

16.36 LIMITS: A package for finding limits

This package loads automatically.

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LIMITS is a fast limit package for REDUCE for functions which are continuous except for computable poles and singularities, based on some earlier work by Ian Cohen and John P. Fitch. The Truncated Power Series package is used for non-critical points, at which the value of the function is the constant term in the expansion around that point. l'Hôpital's rule is used in critical cases, with preprocessing of $\infty - \infty$ forms and reformatting of product forms in order to apply l'Hôpital's rule. A limited amount of bounded arithmetic is also employed where applicable.

16.36.1 Normal entry points

`LIMIT` (*<EXPRN:algebraic>*, *<VAR:kernel>*, *<LIMPOINT:algebraic>*) : *algebraic*

This is the standard way of calling limit, applying all of the methods. The result is the limit of EXPRN as VAR approaches LIMPOINT.

16.36.2 Direction-dependent limits

`LIMIT!+` (*<EXPRN:algebraic>*, *<VAR:kernel>*, *<LIMPOINT:algebraic>*) : *algebraic*

`LIMIT!-` (*<EXPRN:algebraic>*, *<VAR:kernel>*, *<LIMPOINT:algebraic>*) : *algebraic*

If the limit depends upon the direction of approach to the LIMPOINT, the functions `LIMIT!+` and `LIMIT!-` may be used. They are defined by:

$$\begin{aligned} \text{LIMIT!+ } (\text{LIMIT!-}) (\text{EXP,VAR,LIMPOINT}) &\rightarrow \text{LIMIT}(\text{EXP}^*,\epsilon,0), \\ \text{EXP}^* &= \text{sub}(\text{VAR}=\text{VAR}+(-)\epsilon^2,\text{EXP}) \end{aligned}$$